

Worcester County Mathematics League

Varsity Meet 4 - April 13, 2011

Round 2: Algebra 1 - Open



All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Rita has two more than three times the number of Silly Bandz that Bob has. If Rita gave Bob 12 of her Silly Bandz, they would each then have the same number of Silly Bandz. How many Silly Bandz does Rita have?

2. Find the ordered pair solution (x, y) to following system of equations:

$$\frac{2}{x} + \frac{3}{y} = 4$$

$$\frac{1}{x} + \frac{5}{y} = 6$$

3. Six quarts of water are added to 4 quarts of pure antifreeze to fill a 10-quart capacity radiator. Three quarts are then drained off and completely replaced by pure antifreeze. How many quarts of the solution in the radiator must now be removed and completely replaced with pure antifreeze in order to make the solution in the radiator exactly 65% antifreeze?

ANSWERS

(1 pt.) 1. _____ Silly Bandz

(2 pts.) 2. (_____, _____)

(3 pts.) 3. _____ quarts

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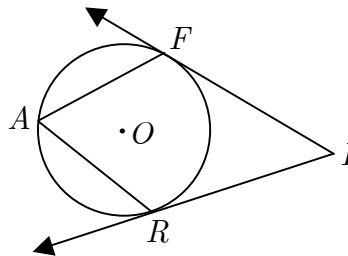
Round 3: Geometry - Open



All answers must be placed in the answer section at the bottom

NO CALCULATOR ALLOWED _____ The diagram is NOT drawn to scale

1. In the diagram below, \overline{IF} and \overline{IR} are tangent to circle O at points F and R . If $\angle FIR = 76^\circ$ and point A is on circle O , find $\angle FAR$ (in degrees).



2. A particle starts at the point $(2, 5)$, moves in a straight line, bounces off the x-axis (so that its angle of reflection is equal to its angle of incidence), and continues in a straight line until it passes through the point $(8, 3)$. Find the x-coordinate of the point at which the particle bounces off of the x-axis.
3. The combined volume of two cubes is numerically equal to the combined length of all of their edges. If the lengths of the edges are integers, compute the volume of the larger cube.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. $x =$ _____

(3 pts.) 3. _____

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Round 4: Logarithms, Exponents and Radicals

4

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Find the positive real value of x that satisfies the equation $\log_x 4 = 4$. Please express your answer in simplest radical form.

2. If $A = \frac{1 + \sqrt{3}}{2\sqrt{2}}$, $B = \frac{-1 + \sqrt{3}}{2\sqrt{2}}$ and $C = \frac{\sqrt{3}}{2}$, compute the value of $\frac{A^2 + B^2 - C^2}{2AB}$.

3. Find all of the values of x that satisfy the equation $\frac{x^{\frac{8}{x}}}{x} = x(x^x)$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 5: Trigonometry - Open



All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Simplify the following expression in terms of a single trigonometric function without fractions and without negative exponents:

$$\frac{\cot A(1 + \tan^2 A)}{\tan A}$$

2. Find all of the values of x , $0^\circ \leq x < 360^\circ$, that satisfy the equation

$$3 \sin^2 x = 1 - \frac{5}{2} \cos x.$$

3. Find the value of x such that $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{x}{325}\right)$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places, unless stated otherwise (2 pts. each)

APPROVED CALCULATORS ALLOWED

1. The number 1102 is called “triprimate” because it is the product of exactly three distinct prime numbers ($1102 = 2 \times 19 \times 29$). How many positive integers less than 300 are triprimate numbers?
2. The average speed of the current in the Muddy River is 2 miles per hour. In still water Tom can paddle his canoe at an average speed of 5 mph. If there are 12 hours of daylight on March 21st, and Tom starts at daybreak of that day, how many miles can he paddle upstream and still be able to return to the same location before nightfall?
3. To the nearest second, what time after 3:00 will the hour hand and minute hand of a standard analog clock first form a right angle? On the answer sheet, record your answer as H:MM:SS, where H, M and S represent the hour, minutes, and seconds, respectively, for the time.
4. If a and b are positive real numbers such that $\log_a b^2 + \log_b a^2 = 5$, then b can be written in terms of a in two distinct ways. Find both expressions in simplest form, using simplest radical form if necessary.
5. Let A , B , C , and D be integers. If $\frac{\sin 4x}{\cos x} = A + B \sin x + C \sin^2 x + D \sin^3 x$, find the ordered four-tuple (A, B, C, D) .
6. A circular sheet of paper of radius 6 inches is cut into three identical sectors. If each sector is formed into a cone (with no overlap), what is the height of each cone (in inches)? Please write your answer in simplest radical form.
7. Compute the unit’s digit of the sum $1^5 + 2^5 + 3^5 + 4^5 + 5^5 + \dots + 2010^5 + 2011^5$.
8. How many ordered triples (x, y, z) of integers satisfy the following set of equations?
$$\begin{aligned}x + yz &= 6 \\y + xz &= 6 \\z + xy &= 6\end{aligned}$$
9. Each of the numbers 228, 344, and 518 leave the same remainder when divided by the positive integer N . Compute the maximum possible value of N .

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ANSWERS

Round 1

1. 2640
2. 2
3. 2.321 (or $2.321_{\bar{5}}$)

Round 2

1. 35
2. $\left(\frac{7}{2}, \frac{7}{8}\right) = \left(3\frac{1}{2}, \frac{7}{8}\right) = (3.5, 0.875)$ (or equivalent)
3. $\frac{5}{3} = 1\frac{2}{3} = 1.\bar{6}$

Round 3

1. 52° or 52
2. $\frac{23}{4} = 5\frac{3}{4} = 5.75$
3. 64

Round 4

1. $\sqrt{2}$ (only)
2. $\frac{1}{2} = 0.5$
3. -4, 1, 2 (need all 3, any order)

Round 5

1. $\csc^2 A = (\csc A)^2$
2. 120° , 240° (or 120, 240 & need both, either order)
3. 165

Team Round

1. 32
2. $\frac{126}{5} = 25\frac{1}{5} = 25.2$
3. 3:32:44
4. a^2 and \sqrt{a} (need both, either order)
5. (0, 4, 0, -8) (only)
6. $4\sqrt{2}$
7. 6
8. 5
9. 58